

# Warsaw University of Technology | Doctoral School No. 3

Course offered in the Doctoral School No. 3  
– Spring semester of the 2021/2022 academic year

TITLE
Queueing Theory
CONDUCTING UNIT
Doctoral School No. 3
SCIENTIFIC DISCIPLINE
Information and communication technology
IMPLEMENTING UNIT
103000 - Faculty of Electronics and Information Technology
FULL DESCRIPTION
<p>The proposed course is devoted to queueing theory, a fundamental approach to performance analysis of service systems whose behavior is driven by events of probabilistic nature. In particular, the mathematical tools provided by queueing theory are necessary to calculate performance measures of computer systems or communication networks where jobs or packets, respectively, arrive at the system in a stochastic manner and their service times are governed by probability distributions. Typical examples of metrics related to such measures are the average job processing time in the computer system or the average packet delay time in the communication network. Needless to say, evaluating performance measures is of utmost importance to the proper design and operation of service systems.</p> <p>The aim of the course is to familiarize participants with the concepts and methods of queueing theory that are applicable to the analysis of the ICT-related service systems, ranging from isolated systems (often called queueing systems, e.g., single-server, single-queue system) to networks of queueing systems. It should be noted that fields of applications of queueing theory are much wider than ICT. Examples of such fields are transportation and road traffic, logistics, and production scheduling.</p> <p>The presentation will be divided into two parts: analytical (two-third of the course) and simulation (on-third).</p> <p>Part I will start with the memoryless queueing systems (where both client inter-arrival times and client service times are described by the exponential distribution) that can be efficiently analyzed using Markov chains. The memoryless queueing systems considered will include both single and multiple server/queue cases and the priorities assigned to different classes of users. Next, we will proceed to queueing systems where the memoryless property is dropped either for inter-arrival times or service times (but not for both); for analyzing such systems the so called embedded Markov chains will be</p>

used. The considerations of queueing systems will conclude with how to treat a single-server, single queueing system in which both the inter-arrival and service times are described by general probability distributions. At the end of the analytical part of the course, queueing networks will be considered. In such networks, every client (e.g., job or packet) from a given stream must traverse a specific path in the network while being served sequentially by the queueing system at each node of the path. Here, so called Jackson networks composed of memoryless single-server, single-queue systems will be studied.

Part II will explain the probabilistic concepts underlying the simulation approach and the basic knowledge of writing simulation programs. The importance of simulation in analyzing the performance of queueing systems (and queueing networks) is due to the fact that it is virtually impossible to analytically calculate performance measures for complex systems encountered in practice. In such cases, simulation programs are used to obtain, unfortunately in a time-consuming way, estimates of the required performance measures. In essence, the simulation run reproduces (using a random number generator) a single realization of a stochastic process that models a given queueing system (or network) to obtain confidence intervals for the values of the selected performance measures.

Both analytical and simulation considerations will be illustrated with examples, and selected examples will be discussed in detail during the exercises and projects.

Although the material presented during the course will be self-contained and in particular it will include explanation of all more advanced probabilistic notions used (for example stochastic processes, including Markov processes), the participants should have a good understanding of basic notions of probabilistic theory (for example random variables and probability distributions) and calculus.

## LITERATURE

L. Kleinrock, Queueing Systems, Volume 1: Theory, Wiley, 1975  
D. Gross, C.M. Harris: Fundamentals of Queueing Theory, Wiley, 1998  
H. Kobayashi: Modeling and Analysis: An introduction to System Performance Evaluation Methodology, Addison-Wesley, 1978  
H. Akimaru, K. Kawashima, Teletraffic: Theory and Applications, Springer-Verlag, 1999  
R. B. Cooper: Introduction to Queueing Theory, 1981

## LEARNING OUTCOMES

After completing the course, participants are expected to know:

1. The basic notions and analytical methods of queueing theory for the memoryless queueing systems  $M/M/k, m$  (described using birth-and-death processes, a subclass of Markov chains), for the systems  $M/G/1$  and  $GI/M/1$ , and for the systems  $M/M/1$  and  $M/G/1$  with priorities.
2. The basic theoretical results for the above systems (Little's equality, Burke's theorem, Pollaczek-Khinchin formula).
3. How to calculate state probabilities and other performance measures for a particular  $M/M/k, m$  system.
4. How to apply discrete-event simulation to a particular  $GI/G/m, k$  system.

## ASSESSMENT METHODS AND CRITERIA; COURSE COMPLETION FORM

Final test and evaluation of individual projects.

LANGUAGE OF THE COURSE		ECTS CREDITS
English		4
TYPE OF CLASSES	NUMBER OF HOURS	COURSE INSTRUCTOR
Lecture	30	Michał Pióro, prof. dr hab. inż.; Artur Tomaszewski, dr hab. inż., prof. uczelni
Tutorials	15	Michał Pióro, prof. dr hab. inż.; Artur Tomaszewski, dr hab. inż., prof. uczelni; Piotr Gajowniczek, dr inż.; Andrzej Bąk, dr inż.
Project	15	Michał Pióro, prof. dr hab. inż.; Artur Tomaszewski, dr hab. inż., prof. uczelni; Piotr Gajowniczek, dr inż.; Andrzej Bąk, dr inż.